The effects of cross-sectional asymmetry and anisotropy of the pore space on double-PFG MR signal

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INTRODUCTION

The double pulsed field gradient (double-PFG) MR sequence (Fig. 1) is a promising diffusion MR method that could be useful in probing restricted geometries using low to moderate gradient strengths. To utilize this technique, one should understand how the double-PFG signal is influenced by pore morphology. Several theoretical [1,2] and numerical [3] studies have been performed where the compartments were taken to have simple shapes such as spherical, cylindrical, and ellipsoidal. All these shapes have one common feature: they are symmetric about their center of gravity.

The main goal of this study is to understand the effect of pore asymmetry on the double-PFG signal. This is important because in white-matter, the axons are typically modeled as cylinders with perfectly circular cross sections. To understand the effects of any anisotropy and asymmetry in the cross sections, we consider ensembles of infinitely long hollow compartments whose cross sections are defined by circular sectors as shown in Figure 2. The size of the sectors is determined by the radius a, and the angle α, where the case α=2π corresponds to the case of circular cylinders. The gradients are assumed to be applied on the plane of these sectors, where the angle between them is denoted by ψ.

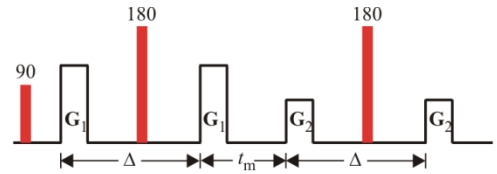

Figure 1. The double-PFG MR pulse sequence involves two pairs of gradient vectors where the time delay between the two pairs is called the mixing time, t_m.

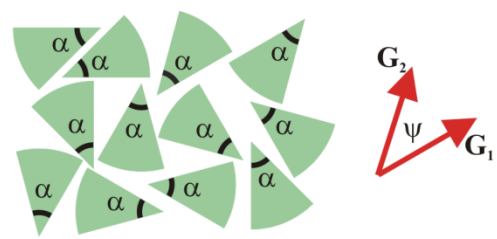

Figure 2. The specimen considered in this study includes a distribution of parallel tubes with randomly oriented asymmetric cross sections, where each compartment's cross section is a sector of a circle defined by the angle α. The two gradients of the double-PFG sequence are applied on the same plane with the sectors.

THEORY and RESULTS

Using an analysis similar to that in [2], we obtained explicit expressions for the double-PFG signal in the narrow pulse and long diffusion time (Δ) regime for this specimen. The expression for long mixing time ($D_0 t_m >> a^2$, where D_0 is the bulk diffusivity), is given by

$$E_\infty \approx 1 - 2(\pi q a)^2 \left(1 - \frac{32}{9\alpha^2}\sin^2(\alpha/2)\right) + (\pi q a)^4 \left[\frac{11}{6} - \frac{512}{45\alpha^2}\sin^2(\alpha/2) - \frac{32}{9\alpha^3}\sin\alpha + \frac{1024}{81\alpha^4}\sin^4(\alpha/2) + \cos 2\psi \left(\frac{\sin^2\alpha}{2\alpha^2} + \frac{512}{81\alpha^4}\sin^4(\alpha/2)\right)\right]$$

while the signal at vanishing mixing times is

$$E_0 \approx 1 - (\pi q a)^2 \left(1 - \frac{32}{9\alpha^2}\sin^2(\alpha/2)\right)(2 + \cos\psi)$$

where $q=(2\pi)^{-1}\gamma\delta G$ with $G=|\mathbf{G}_1|=|\mathbf{G}_2|$.

It is clear from the first equation that a (cos 2ψ) factor emerges, which is responsible for the characteristic curves in the presence of compartment shape anisotropy. Moreover, in the case of full circles, i.e., α=2π, this factor vanishes as the cross sections are isotropic. Figure 3 shows the long mixing time curves for different values of α.

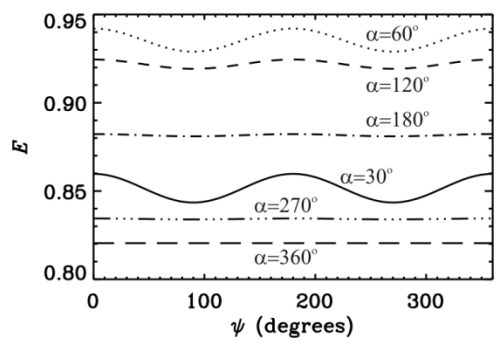

Figure 3. The double-PFG signal intensity plotted against the angle between the two gradients for varying values of α .

The second equation reveals the (2+cos ψ) factor, which is responsible for the bell-shaped functions in the E vs. ψ plots at t_m=0, characteristic of the presence of microscopic anisotropy, which is introduced by the boundaries restricting diffusion. As expected, this term prevails even at α=2π.

The expression for the short mixing time regime has similar qualitative features with that for symmetric pore shapes. Therefore, one may be tempted to assume a circular pore shape if any data revealing compartment shape anisotropy is absent. In this case, one can talk about an "apparent" compartment size that is attainable from such an analysis. Figure 4 shows this apparent pore size against an "expected" pore size, $a_{exp}=a(\alpha/(2\pi))^{1/2}$, which is the radius of a circle whose area is equal to the area of the sector considered. Note that this quantity is proportional to the square root of α. It is clear that the error introduced by the assumption of a circular cross section is relatively small (<10%) for a very large range of α values.

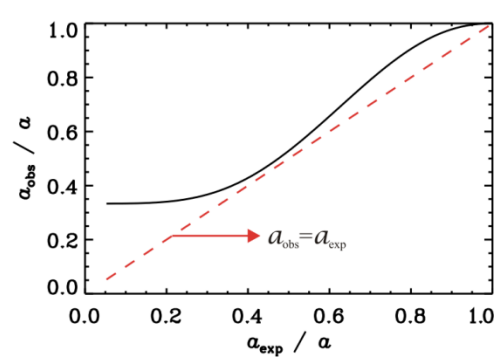

Figure 4. The pore radius observed under the assumption of circular cross sections against the radius of the circle that provides the correct value for the cross sectional area.

CONCLUSION

A wedge-shaped geometry for the cross section of a cylindrical pore was considered to assess the effects of any anisotropy and asymmetry in the cross sections. The findings suggest that a much simpler model of cylinders with isotropic cross sections can be used in representing axons.

References:
[1] Ozarslan and Basser, J Chem Phys,128:154511, 2008. [2] Ozarslan, J Magn Reson, 199:56-67, 2009.
[3] Koch and Finsterbusch, Magn Reson Med, 62:247-254, 2009.